EXTREME FLUCTUATIONS IN SMALL-WORLD-COUPLED AUTONOMOUS SYSTEMS WITH RELAXATIONAL DYNAMICS*

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Synchronization is a fundamental problem in natural and artificial coupled multi-component systems. We investigate to what extent small-world couplings (extending the original local relaxational dynamics through the random links) lead to the suppression of extreme fluctuations in the synchronization landscape of such systems. In the absence of the random links, the steady-state landscape is "rough" (strongly desynchronized state) and the average and the extreme height fluctuations diverge in the same power-law fashion with the system size (number of nodes). With small-world links present, the average size of the fluctuations becomes finite (synchronized state). For exponential-like noise the extreme heights diverge only logarithmically with the number of nodes, while for power-law noise they diverge in a power-law fashion. The statistics of the extreme heights are governed by the Fisher–Tippett–Gumbel and the Fréchet distribution, respectively. We illustrate our findings through an actual synchronization problem in parallel discrete-event simulations.

Keywords: Extreme-value statistics; synchronization; small-world networks; scalable computing.

1. Introduction

Many of our important technological, information, and infrastructure systems form complex networks [1–6] with a large number of components. These networks consist of nodes (components of the system) and links connecting the nodes. The links facilitate some kind of effective interaction/dynamics between the nodes. Examples (with the processes inducing the interaction between the nodes) include high-performance scalable parallel or grid-computing networks (synchronization protocols for massive parallelization) [6], diffusive load-balancing schemes (relocating jobs among processors) [7], the Internet (protocols for sending/receiving packets) [8], or the electric power grid (generating/transmitting power between generators and buses) [5]. Many of these systems are autonomous (by design or historical evolution), i.e., they lack a central regulator. Thus, fluctuations in the "load" in the respective network

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(data/state savings or task allocation in parallel simulations, traffic in the Internet, or voltage/phase in the electric grid) are determined by the collective result of the individual decisions of many interacting "agents" (nodes). As the number of processors on parallel architectures increases to hundreds of thousands [9], grid-computing networks proliferate over the Internet [10, 11], or the electric power-grid covers, e.g., the North-American continent [5], fundamental questions on the corresponding dynamical processes on the respective underlying networks must be addressed.

Typically, large fluctuations in the above networks are to be avoided (e.g., for scalability or stability reasons). In the absence of global intervention or control, this can be a difficult task. Motivated by a recent example [6] for small-world (SW) [12] synchronized autonomous systems in the context of scalable parallel computing, we investigate the steady-state properties of the extreme fluctuations in SW-coupled interacting systems with relaxational dynamics [13]. Since the introduction of SW networks [12] it has been well established that such networks can facilitate autonomous synchronization [14–16]. In addition to the average "load" in the network, knowing the typical size and the distribution of the extreme fluctuations [17–19] is of great importance from a system-design viewpoint, since failures and delays are triggered by extreme events occurring on an individual node.

Relationship between extremal statistics and universal fluctuations in correlated systems has been studied intensively [20–35]. The focus of a number of these studies was to find connections, if any, between the probability distribution of *global* observables or order parameters (such as the width in surface growth problems [36] or the magnetization in magnetic systems [37]) and known universal extreme-value limit distributions [17–19]. Recent analytic results demonstrated [28, 31] that, in general (except for special cases [27]), there are no such connections. Here we discuss to what extent SW couplings (extending the original dynamics through the random links) lead to the suppression of the extreme fluctuations of the *local* order parameter or field variable in various noisy environments. We illustrate our findings on an actual synchronization problem in scalable parallel computing [6]. In Sec. 2 we review the well-known extreme-value limit distributions for exponential-like and power-law-tail distributed random variables. In Sec. 3 we review recent results [13] on the scaling behavior of the extreme fluctuations and their distribution. for the Edwards–Wilkinson model [38] on SW networks [39] with exponential-like noise. In Sec. 4 we apply these results to study the extreme load fluctuations in SW-synchronized parallel discrete-event simulation (PDES) schemes [40, 41], applicable to high performance parallel architectures and large-scale grid-computing networks. In Sec. 5 we extend our earlier studies [13] and consider the synchronization problem in the presence of power-law tailed noise. We finish the Letter with a brief summary in Sec. 6.

2. A Brief Review of the Extreme-Value Limit Distribution for Independent Random Variables

2.1. *Exponential-like variables*

First, we consider the case when the individual complementary cumulative distribution $P_>(x)$ (the probability that the individual stochastic variable is greater than x) decays faster than any power law, i.e., exhibits an exponential-like tail in the asymptotic large-x limit. (Note that in this case the corresponding probability density function displays the same exponential-like asymptotic tail behavior.) We will assume $P_>(x) \simeq e^{-cx^\delta}$ for large x values, where c and δ are constants. Then the cumulative distribution $P_<^{\max}(x)$ for the largest of the N events (the probability that the maximum value is less than x) can be approximated as [32, 42, 43]

$$P_<^{\max}(x) = [P_<(x)]^N = [1 - P_>(x)]^N = e^{N\ln[1-P_>(x)]} \simeq e^{-NP_>(x)} , \tag{1}$$

where one typically assumes that the dominant contribution to the statistics of the extremes comes from the tail of the individual distribution $P_>(x)$. With the exponential-like tail in the individual distribution, this yields

$$P_<^{\max}(x) \simeq e^{-e^{-cx^\delta + \ln(N)}} . \tag{2}$$

The extreme-value limit theorem states that there exists a sequence of scaled variables $\tilde{x} = (x - a_N)/b_N$, such that in the limit of $N \to \infty$, the extreme-value probability distribution for $\tilde{x}$ asymptotically approaches the Fisher–Tippett–Gumbel (FTG) distribution [17, 18]:

$$\tilde{P}_<^{\max}(\tilde{x}) \simeq e^{-e^{-\tilde{x}}} , \tag{3}$$

with mean $\langle\tilde{x}\rangle = \gamma$ ($\gamma = 0.577\ldots$ being the Euler constant) and variance $\sigma_{\tilde{x}}^2 = \langle\tilde{x}^2\rangle - \langle\tilde{x}\rangle^2 = \pi^2/6$. From Eq. (2), one can deduce [43, 44] that to leading order the scaling coefficients are $a_N = \left[\frac{\ln(N)}{c}\right]^{1/\delta}$ and $b_N = (\delta c)^{-1}\left[\frac{\ln(N)}{c}\right]^{(1/\delta)-1}$. The average value of the largest of the N original variables then scales as

$$\langle x_{\max}\rangle = a_N + b_N\gamma \simeq \left[\frac{\ln(N)}{c}\right]^{1/\delta} \tag{4}$$

(up to $\mathcal{O}(\frac{1}{\ln(N)})$ correction) in the asymptotic large-N limit. When comparing with experimental or simulation data, instead of Eq. (3), it is often convenient to use the form of the FTG distribution which is scaled to zero mean and unit variance, yielding

$$\tilde{P}_<^{\max}(y) = e^{-e^{-(ay+\gamma)}} , \tag{5}$$

where $a = \pi/\sqrt{6}$ and γ is the Euler constant. In particular, the corresponding FTG density then becomes

$$\tilde{p}^{\max}(y) = ae^{-(ay+\gamma) - e^{-(ay+\gamma)}} . \tag{6}$$

2.2. ***Power-law tailed variables***

Now consider independent identically distributed random variables where the tail of the complementary cumulative distribution decays in a power law fashion, i.e., $P_>(x) \simeq A/x^\mu$ for large values of x. Assuming again that the dominant contribution to the statistics of the extremes comes from the tail of the individual distribution [32, 42, 43], Eq.(1) yields

$$P_<^{\max}(x) \simeq e^{-NP_>(x)} \simeq e^{-NA/x^\mu} . \tag{7}$$

Introducing the scaled variable $\tilde{x} = x/b_N$, where $b_N = (AN)^{1/\mu}$, yields the standard form of the so called Fréchet distribution for the extremes in the asymptotic large-N limit [17, 19]

$$\tilde{P}^{\max}_{<}(\tilde{x}) = e^{-1/\tilde{x}^{\mu}} \ , \tag{8}$$

and the corresponding probability density

$$\tilde{p}^{\max}(\tilde{x}) = \frac{\mu}{\tilde{x}^{\mu+1}} e^{-1/\tilde{x}^{\mu}} \ . \tag{9}$$

One can note that the tail behavior of the extremes has been inherited from that of the original individual variables, i.e., $\tilde{p}^{\max}(\tilde{x}) \sim 1/\tilde{x}^{\mu+1}$ for large values of $\tilde{x}$. The first moment of the extreme exist if $\mu > 1$ and for the average value of the largest of the N original power-law variables one finds

$$\langle x_{\max} \rangle = b_N \langle \tilde{x} \rangle \simeq \Gamma(1-1/\mu)(AN)^{1/\mu} \sim N^{1/\mu} \tag{10}$$

where $\Gamma(z)$ is Euler's gamma function. For comparison with experimental or simulation data it is often convenient to use an alternative scaling for the extremes $y = x/\langle x_{\max} \rangle$, yielding collapsing ($N$-independent) probability density functions similar to Eq.(9)

$$\tilde{p}^{\max}(y) = \frac{\mu}{\Gamma^{\mu}(1-1/\mu) y^{\mu+1}} e^{-1/(\Gamma^{\mu}(1-1/\mu) y^{\mu})} \ . \tag{11}$$

3. Extreme Fluctuations in the Small-World-Coupled Edwards–Wilkinson Model

We consider the simplest stochastic model with linear relaxation on a SW network,

$$\partial_t h_i(t) = -(2h_i - h_{i+1} - h_{i-1}) - \sum_{j=1}^{N} J_{ij}(h_i - h_j) + \eta_i(t) \ , \tag{12}$$

where $h_i(t)$ is the local height or field variable at node i at time t and $\eta_i(t)$ is a delta-correlated short-tailed (e.g., Gaussian) noise. The symmetric matrix J_{ij} (with matrix elements being equal to 0 or p) represents the quenched random links of strength p on top of a one dimensional regular lattice. In the construction of the SW network presented here, each node has exactly one random neighbor [Fig. 1(a)]. That is, pairs of nodes are selected at random, and once they are linked, they cannot be selected again. This construction is motivated by our application [6, 45] to scalable PDES schemes (see Sec. 4), where fluctuations in the individual degree of the nodes are to be avoided. Our construction of the SW network differs from both the original ("rewiring") [12,14] and the "soft" version [39,46–49] of the SW network (where an Erdős–Rényi random graph [50] is thrown on top of a regular lattice). Our construction too, however, exhibits a well balanced coexistence among short- and long-range links (random links are placed on the top of a regular substrate). Further, the average path length $\langle l \rangle_N$ (the average minimum number of links connecting two randomly chosen nodes) scales *logarithmically* with the system size N [Fig. 1(b)], i.e., like most other random networks [1], it too exhibits the "small-world" property (or low-degree of separation).

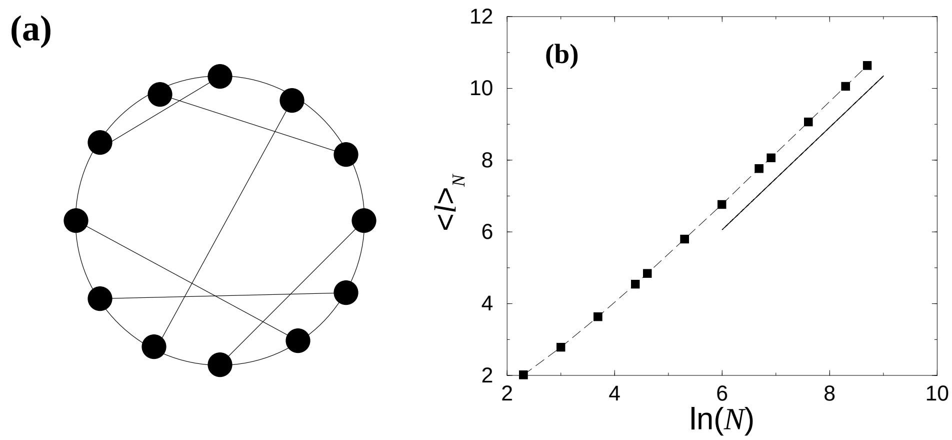

Fig. 1. (a) A small-world network where random links are added to the ring, such that each node has exactly one random link. (b) Average shortest path as a function of the logarithm of the number of nodes for our small-world synchronization network shown in (a). The straight line represents the slope of the asymptotic large N behavior of the average shortest path $\langle l\rangle_N \simeq 1.42\ln(N)$.

Equation (12), the extension of the the Edwards–Wilkinson (EW) model to a SW "substrate", where the strength of the interactions through the random links is p, is a prototypical synchronization problem with "local" relaxation. The width

$$w \equiv \sqrt{\left\langle \frac{1}{N}\sum_{i=1}^{N}(h_i-\bar{h})^2 \right\rangle}\ , \tag{13}$$

borrowing the framework from non-equilibrium surface-growth phenomena, provides a sensitive measure for the average degree of synchronization in coupled multi-component systems [6, 51]. In Eq. (13) $\langle\ldots\rangle$ denotes an ensemble average over the noise in Eq. (12), and $\bar{h}{=}(1/N)\sum_{i=1}^{N} h_i$ is the mean height. In addition to the width, we will study the scaling behavior of the largest fluctuations (e.g., above the mean) in the steady-state

$$\langle\Delta_{\max}\rangle \equiv \langle h_{\max}-\bar{h}\rangle\ . \tag{14}$$

Equation (12) (and its generalization with a Kardar–Parisi–Zhang (KPZ)-like non-linearity [52]) is also believed to govern the steady-state progress and scalability properties of a large class of PDES schemes [6, 51, 53–55]. In this context, the local height variables $\{h_i(t)\}_{i=1}^{N}$ correspond to the progress of the individual processors after t parallel steps (Sec. 4). The EW/KPZ-type relaxation at a coarse-grained level originates from the "microscopic" (node-to-node) synchronizational rules. In the absence of the random links with purely short-range connections, the corresponding steady-state landscape is rough [36] (de-synchronized state), i.e., it is dominated by large-amplitude long-wavelength fluctuations. The extreme values of the local fluctuations emerge through these long-wavelength modes and, in one dimension, the extreme and average fluctuations follow the *same* power-law divergence with the system size [20, 34, 35, 54, 55]

$$\langle\Delta_{\max}\rangle \sim w \sim N^{\alpha} , \tag{15}$$

where α is the roughness exponent [36] [Figs. 2(a) and 3(a)]. The diverging width is related to an underlying diverging lengthscale, the lateral correlation length, which reaches the system size N for a finite system. In PDES schemes the average local memory requirement on each node is proportional to the spread of the progress of the individual processors (the width of the landscape of the progress of the simulation). Thus, a diverging width (strongly de-synchronized state) [Fig. 2(a)] can seriously hinder scalable data management [54, 55], motivating the implementation of a SW synchronization network [6] (Sec. 4).

The important feature of the EW model on SW networks is the development of an effective nonzero mass $\Sigma(p)$, corresponding to an actual or pseudo gap in a field theory sense [39, 48, 56], generated by the quenched-random structure [39]. In turn, both the *average* correlation length $\xi \simeq [\Sigma(p)]^{-1/2}$ and the width $w \simeq (1/\sqrt{2})[\Sigma(p)]^{-1/4}$ approach a finite value (synchronized state) and become self-averaging in the $N\to\infty$ limit [45]. For example, for our above described construction of the SW network [39], for small p values, $\Sigma(p) \sim p$. Thus, the average correlation length becomes *finite* for an arbitrarily small but nonzero strength of the random links (one such link per site). This is the fundamental effect of extending the original dynamics to a SW network: it decouples the fluctuations of the originally correlated system. Then, the extreme-value limit theorems can be applied using the number of independent blocks N/ξ in the system [32, 43]. Further, if the tail of the noise distribution decays in an exponential-like fashion, the individual relative height distribution will also do so [57], and depends on the combination Δ_i/w, where $\Delta_i = h_i - \bar{h}$ is the relative height measured from the mean at site i. Considering, e.g., the fluctuations above the mean for the individual sites, we will then have $P_{>}(\Delta_i) \simeq \exp[-c(\Delta_i/w)^{\delta}]$, where $P_{>}(\Delta_i)$ denotes the "disorder-averaged" (averaged over network realizations) single-site relative height distribution, which becomes independent of the site i for SW networks. From the above it follows that the cumulative distribution for the extreme-height fluctuations relative to the mean $\Delta_{\max} = h_{\max} - \bar{h}$, if scaled appropriately, will be given by Eq. (3) [or alternatively by Eq. (5)] in the asymptotic large-N limit (such that $N/\xi \gg 1$). Further, from Eq. (4), the average maximum relative height will scale as

$$\langle\Delta_{\max}\rangle \simeq w \left[\frac{\ln(N/\xi)}{c}\right]^{1/\delta} \simeq \frac{w}{c^{1/\delta}} [\ln(N)]^{1/\delta} , \tag{16}$$

where we kept only the leading order term in N. Note, that both w and ξ approach their *finite* asymptotic N-independent values for SW-coupled systems. Also, the same logarithmic scaling with N holds for the largest relative deviations below the mean $\langle\bar{h} - h_{\min}\rangle$ and for the maximum spread $\langle h_{\max} - h_{\min}\rangle$. This weak logarithmic divergence, which one can regard as marginal, ensures synchronization for practical purposes in SW coupled multi-component systems with local relaxation in an environment with exponential-like noise.

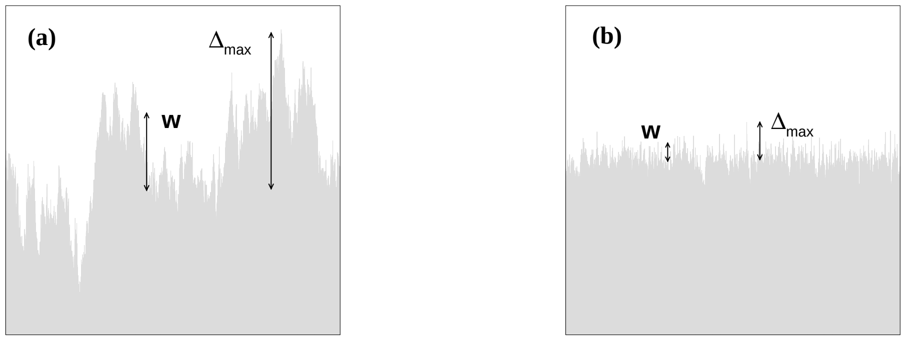

Fig. 2. Snapshots of the virtual time horizon for the conservative PDES scheme with $N{=}10^4$ processors in the steady state. (a) The processors are connected in a ring-like fashion; (b) the processors are connected by a small-world topology and the additional synchronization through the random link is performed with probability $p{=}0.10$ at every parallel step. The vertical scales indicating the progress of the individual processing elements are the same in (a) and (b). The arrows indicate the average (w) and the extreme ($\Delta_{\max}$) fluctuations in virtual time horizon.

4. Application to Scalable Parallel Discrete-Event Simulations on High-Performance Parallel and Grid Computing Networks

Developing and implementing massively parallel algorithms is among the most challenging areas in computer science and in computational science and engineering [58]. While there are numerous technological and hardware-related points, e.g., concerning efficient message passing and fast communications between computer nodes, the theoretical algorithmic challenge is often as important. This is particularly true for cases when the parallel algorithm has to simulate the time evolution of a complex system in which the local changes (discrete events) in the configuration are inherently *asynchronous*. The basic notion of the above discrete-event systems is that time is continuous and the changes in the local configurations occur at random instants of time (hence the asynchrony of the time evolution of the local configuration). Between events, the local configuration remains unchanged. In physics or chemistry these types of simulations are most commonly referred to as dynamic or kinetic Monte Carlo simulations [59]. In computer science they are called discrete-event simulations. PDES schemes [40,41,60] are capable of faithfully simulating such systems in a massively parallel fashion. For very large interacting systems (where trivial or "embarrassing" parallelization is not possible or highly inefficient due to CPU/memory limitations), PDES is the only way to perform parallel simulations *without* changing the original underlying asynchronous dynamics. Examples of PDES applications include dynamic channel allocation in cell phone communication networks [61,62], models of the spread of diseases [63], and dynamic phenomena in highly anisotropic magnetic thin films [64–66]. In these examples the discrete events are call arrivals, infections, and changes of the orientation of the local magnetic moments, respectively.

The difficulty of parallel discrete-event simulations is that the local changes (updates) in the system are not synchronized by a global clock. The essence of the corresponding PDES schemes, capable of faithfully simulating these systems, is to algorithmically parallelize "physically" non-parallel dynamics of the underlying systems. This requires some kind of synchronization to ensure causality [40]. The two basic ingredients of PDES schemes are a set of local simulated times (or virtual times [67]) and a synchronization scheme. First, a scalable parallel scheme must ensure that the average progress rate of the simulation approaches a nonzero asymptotic value in the long-time limit as the number of processors (or nodes) goes to infinity. Second, the spread of the virtual time horizon (the spread of the progress of the individual processors) should be bounded as the number of processors goes to infinity [68]. The second requirement is crucial for the measurement phase of the simulation to be scalable: a diverging spread of the virtual time horizon (as the number of processors goes to infinity) hinders scalable data management [54, 55]. Temporarily storing a large amount of simulated data on each node (proportional to the spread of the virtual time horizon) is limited by available memory, while frequent global synchronizations can get computationally costly for a large number of nodes on certain parallel architectures. In the latter case, one aims to devise a parallel scheme where the processors make a nonzero close-to-uniform progress *without global synchronization*. In such a scheme, the processors autonomously learn the global state of the system (without explicit global messages) and adjust their progress rate accordingly [6].

PDES algorithms concurrently advance the local simulated time on each processor [or processing element (PE)], without violating causality. In a "conservative" PDES scheme [69–72], only those PEs that are guaranteed not to violate causality are allowed to process their events and increment their local time. The rest of the PEs must "idle." In an "optimistic" approach [67], the processors do not have to idle, but since causality is not guaranteed at every update, the simulated history on certain processors can become corrupted. This requires a complex "rollback" protocol to correct erroneous computations. Both simulation approaches lead to an evolving and fluctuating time horizon during algorithmic execution. Similar to our earlier results [51] in finding a connection between certain conservative PDES schemes [71, 72] and kinetic roughening in nonequilibrium surfaces [36, 52, 73], a "complex system" approach was also successful to establish the connection [74, 75] between rollback-based (or optimistic) PDES schemes [67] and self-organized criticality [76, 77]. In what follows, we will focus on the synchronizability of conservative PDES schemes, in particular, on the behavior of the width and the largest fluctuations of the virtual time horizon.

Consider an arbitrary one-dimensional system with nearest-neighbor interactions, in which the discrete events (update attempts in the local configuration) exhibit Poisson asynchrony. In the one-site-per-PE scenario, each PE has its own local simulated time, constituting the virtual time horizon $\{h_i(t)\}_{i=1}^{N}$ (essentially, the progress of the individual nodes). Here t is the discrete number of parallel steps executed by all PEs, which is proportional to the wall-clock time and N is the number of PEs. According to the basic conservative synchronization scheme [71, 72], at each parallel step t, only those PEs for which the local simulated time is not greater then the local simulated times of their virtual neighbors, can increment their local time by an *exponentially* distributed random amount. (Without loss of generality

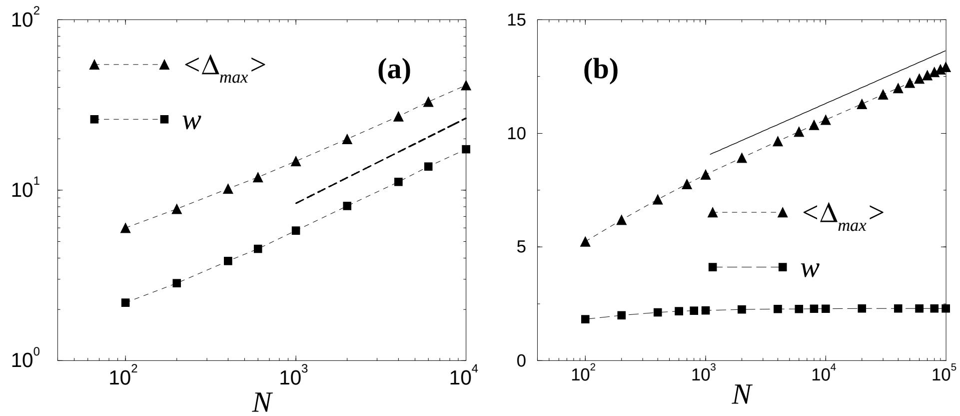

Fig. 3. (a) Scaling behavior of the average (w) and the extreme ($\Delta_{\max}$) fluctuations in the virtual time horizon for the conservative PDES scheme in the steady state. The processors are connected in a ring-like fashion (log-log scales). The dashed line represents the theoretical power law with the roughness exponent α=1/2. (b) The same quantities as in (a), but the processors are connected by a small-world topology and the additional synchronization through the random link is performed with probability p=0.10 at every parallel step (log-normal scales). The solid straight line indicates the weak logarithmic increase of the extreme fluctuations with the system size.

we assume that the mean of the local time increment is one in simulated time units.) Thus, denoting the virtual neighborhood of PE i by S_i, if $h_i(t) \leq \min_{j\epsilon S_i}\{h_j(t)\}$, PE i can update the configuration of the underlying site it carries and determine the time of the next event. Otherwise, it idles. Despite its simplicity, this rule preserves unaltered the asynchronous causal dynamics of the underlying system [71, 72]. In the original algorithm [71, 72], the virtual communication topology between PEs mimics the interaction topology of the underlying system. For example, for a one-dimensional system with nearest-neighbor interactions, the virtual neighborhood of PE i, S_i, consists of the left and right neighbor, PE $i-1$ and PE $i+1$. It was shown [51] that then the virtual time horizon exhibits KPZ-like kinetic roughening and the steady-state behavior in one dimension is governed by the EW Hamiltonian. Thus, both the average (the spread in the progress of the individual PEs) and the extreme fluctuations of the virtual time horizon diverge when $N\to\infty$ [Figs. 2(a) and 3(a)], hindering efficient data collection in the measurement phase of the simulation [54]. To achieve a near-uniform progress of the PEs *without* employing frequent global synchronizations, it was shown [6] that including randomly chosen PEs (*in addition* to the nearest neighbors) in the virtual neighborhood, results in a finite average width [Figs. 2(b) and 3(b)]. Here we demonstrate that SW synchronization in the above PDES scheme results in logarithmically increasing extreme fluctuations in the simulated time horizon, governed by the FTG distribution.

In the SW-synchronized version of the conservative PDES scheme each PE has exactly one random neighbor (in addition to the nearest neighbors) and the local simulated time of the random neighbor is checked only with probability p at every simulation step. Thus, the effective "strength" of the random links is controlled by

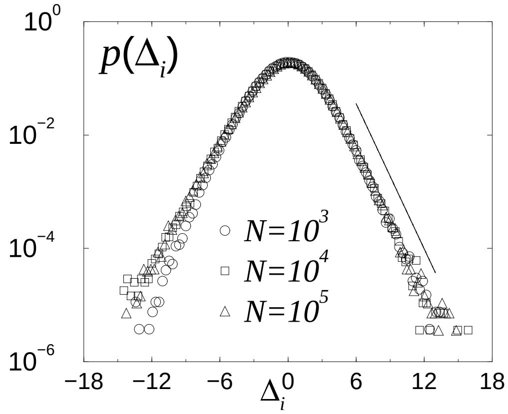

Fig. 4. Disorder-averaged probability densities for the local height fluctuations for the SW-synchronized (p=0.10) landscape for three system sizes indicated in the figure. Note the log-normal scales. The solid straight line indicates the exponential tail.

the relative frequency p of the basic synchronizational steps through those links. Note that the occasional extra checking of the simulated time of the random neighbor is not needed for the faithfulness of the simulation. It is merely introduced to control the width of the time horizon [6].

To study the extreme fluctuations of the SW-synchronized virtual time-horizon, we "simulated the simulations", i.e., the evolution of the local simulated times based on the above exact algorithmic rules [13]. By constructing histograms for Δ_i, we observed that the tail of the disorder-averaged individual relative-height distribution decays exponentially (δ=1) [Fig. 4]. Then, we constructed histograms for the extreme-height fluctuations Fig. 5(a). The scaled histograms, together with the similarly scaled FTG density Eq. (6), are shown in Fig. 5(b). We also observed that the distribution of the extreme values becomes *self-averaging*, i.e., independent of the network realization. Figure 3(b) shows that for sufficiently large N (such that w essentially becomes system-size independent) the average (or typical) size of the extreme-height fluctuations diverge *logarithmically*, according to Eq. (16) with δ=1. We also found that the largest relative deviations below the mean $\langle \bar{h} - h_{\min} \rangle$, and the maximum spread $\langle h_{\max} - h_{\min} \rangle$ follow the same scaling with the system size N. Note, that for our specific system (PDES time horizon), the "microscopic" dynamics is inherently nonlinear, but the effects of the nonlinearities only give rise to a renormalized mass $\Sigma(p)$ (leaving $\Sigma(p)>0$ for all $p>0$) [6]. Thus, the dynamics is effectively governed by relaxation in a small world, yielding a finite correlation length and, consequently, the slow logarithmic increase of the extreme fluctuations with the system size [Eq. (16)]. Also, for the PDES time horizon, the local height distribution is asymmetric with respect to the mean, but the average size of the height fluctuations is, of course, finite for both above and below the mean. This specific characteristic simply yields different prefactors for the extreme fluctuations [Eq. (16)] above and below the mean, leaving the logarithmic scaling with N un-

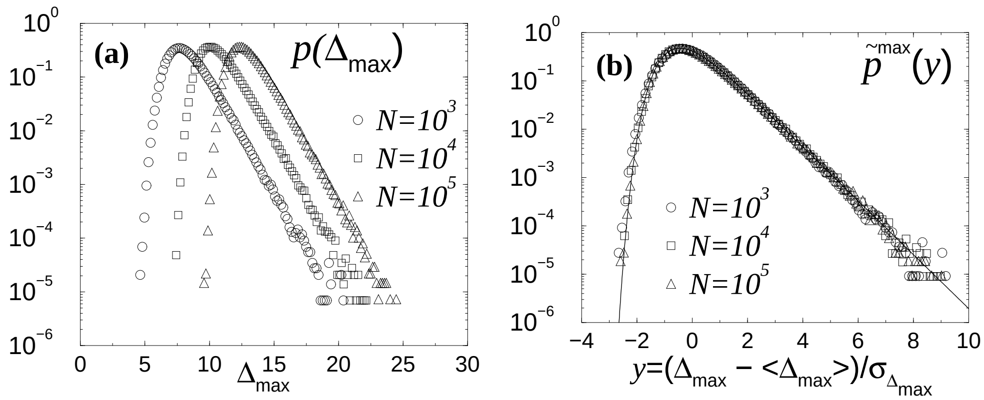

Fig. 5. (a) Disorder-averaged probability densities for the extreme-height fluctuations for the SW-synchronized conservative PDES time horizons with p=0.10 for three system sizes indicated in the figure. Note the log-normal scales. (b) The same as (a) but the probability densities are scaled to zero mean and unit variance. The solid curve corresponds to the similarly scaled FTG density Eq. (6) for comparison.

changed.

5. Synchronization in the Presence of Power-Law Noise

Employing SW-like synchronization networks to suppress large fluctuations was successful in the presence exponential-like "noise". We now investigate the scenario when the noise distribution exhibits a power-law tail. We consider the synchronization problem from parallel discrete-event simulations (see Sec. 4) for power-law tailed asynchrony. The condition for updating the local "height" variables in the synchronization landscape (corresponding to the local virtual times) is *unchanged*, i.e., a node is only allowed to increment its local simulated time h_i if it is a local minimum in the virtual neighborhood (possibly including the random neighbor with probability p). The increment, however, is now drawn from a *power-law* probability density $p(\eta) \sim 1/\eta^{\gamma+1}$. Since the above local update rule is, essentially, relaxation on the network, this model also serves as a prototypical model for relaxation on SW networks in an environment with power-law noise. The above synchronization rules can be applied to simulating systems with non-Poisson asynchrony, relevant to various transport and transmission phenomena in natural and artificial systems [78–80]. For example, in Internet or WWW traffic, in part, as a result of universal power-law tail file-size distributions [81, 82], service times exhibit similarly tailed distributions in the corresponding queueing networks [83–85]. In turn, when simulating these systems, the corresponding PDES should use power-law tail distributed local simulated time increments.

For a purely one-dimensional connection topology (in the absence of the random links) we observed kinetic roughening. Since the time to reach the steady, the relaxation time in the steady state, and the surface width all diverge with the

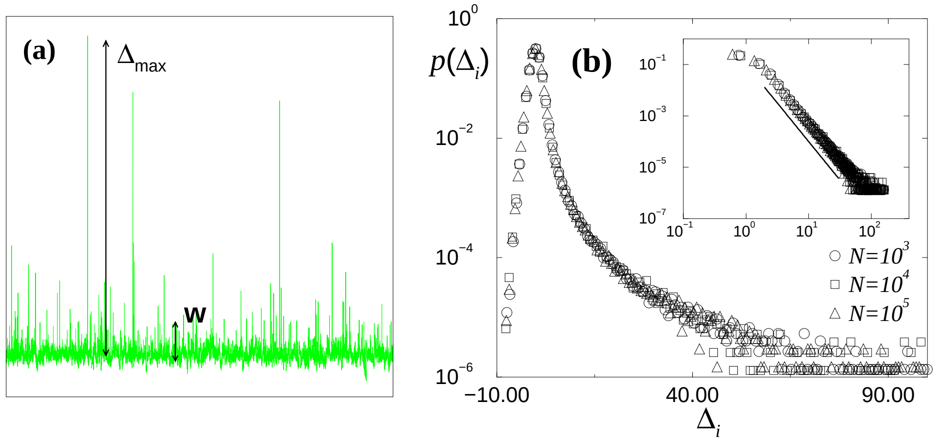

Fig. 6. (a) Snapshot for the SW-synchronized (p=0.10) landscape in a power-law noise environment (γ=3) for N=10^4 nodes. (b) Disorder-averaged probability densities for the same parameters for the local height fluctuations for three system sizes indicated in the figure. Note the log-normal scales. The inset shows the same (for the positive domain) on log-log scales. The solid line corresponds to the slope of μ+1≈3.

number of nodes, it is difficult to measure the roughness exponent accurately. It is well documented [86], however, that KPZ-like growth in the presence of power-law noise leads to anomalous roughening (yielding a roughness exponent greater than 1/2 in one dimension).

Here we show and discuss results for the power-law noise generated growth on SW networks. We have chosen two values of γ governing the tail of the probability density for the noise: γ=3 and γ=5. For both of these cases the noise have a finite mean and variance. One can expect a power-law tail (at least for above the mean) for the probability density of the individual local height fluctuations $p(\Delta_i) \sim 1/\Delta_i^{\mu+1}$, once the noise is "filtered through" the collective dynamics. On Fig. 6(a) we show a snapshot for the resulting synchronization landscape, indicating the presence of some rare but very large fluctuations above the mean. Since the local update rules lead to nonlinear (KPZ-like) effective interactions, we could not predict the exponent of the local height distribution. Instead, we constructed histograms representing $p(\Delta_i)$. For the above two values of the noise exponent, γ=3 and γ=5, we observed power-law tail exponents for $p(\Delta_i) \sim 1/\Delta_i^{\mu+1}$ as well, but with exponents clearly differring from that of the noise, μ≈2 and μ≈4, respectively. Figure 6(b) shows $p(\Delta_i)$ for the former. The figure indicates that for large Δ_i a power-law tail developes, while fluctuations below the mean exhibit an exponential-like tail. This asymmetry is due to the asymmetry in the microscopic update rules: local mimima were *incremented* by power-law distributed random amount, hence anomalously large deviations above the mean can emerge.

Then we analyzed the scaling behavior of the average and the extreme height fluctuations in the associated synchronization landscape. In the limit of large N,

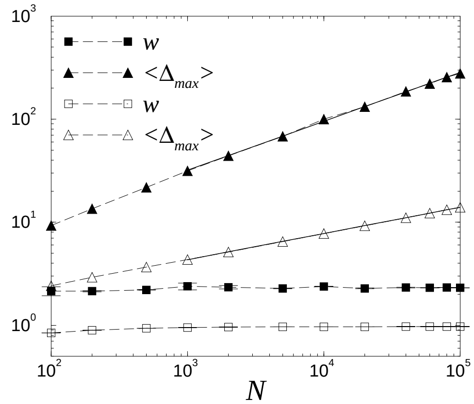

Fig. 7. Steady-state scaling behavior of the average (w) and the extreme ($\Delta_{\max}$) fluctuations in the synchronization landscape for power-law noise with exponent γ=3 (filled symbols) and γ=5 (open symbols) using SW links with p=0.10. The straight solid-line segments are the best-fit power laws ($\langle\Delta_{\max}\rangle \sim N^{1/\mu}$ with $1/\mu$=0.47 and $1/\mu$=0.25, respectively) for the extreme fluctuations for system sizes $N \geq 10^3$, according to Eq. (10).

w becomes system-size independent, while the extreme-height fluctuations above the mean diverge in a power-law fashion according to Eq. (10) [Fig. 7]. Fitting a power law for $N \geq 10^3$ yields $\langle\Delta_{\max}\rangle \sim N^{0.47}$ and $\langle\Delta_{\max}\rangle \sim N^{0.25}$ for the two cases analyzed in Fig. 7, for γ=3 and γ=5, respectively. In order to understand the underlying reason for this divergence, we analyzed the histograms constructed for the probability density of the extreme height fluctuations $p(\Delta_{\max})$ [Fig. 8(a)]. The shapes of these histograms suggest that the limit distribution is of Fréchet type. We constructed the histograms for the scaled variable $y = \Delta_{\max}/\langle\Delta_{\max}\rangle$. Then using $\mu = 1/0.47 = 2.1$ and $\mu = 1/0.25 = 4$ as implied by the scaling behavior of $\langle\Delta_{\max}\rangle$ [Eq. (10)], we plotted the similarly scaled Fréchet density Eq. (11) [Fig. 8(b). These results indicate that the effect of the random links in SW networks is again to decouple the local field variables, an in turn, the statistics of the extremes are governed by the Fréchet distribution. Consequently, the average size of the extremes diverges in a power-law fashion $\langle\Delta_{\max}\rangle \sim N^{1/\mu}$. This power-law divergence is *not* the result of a divergent lengthscale emerging from the cooperative effects of the interacting nodes. On the contrary, the local field variables become effectively independent using SW synchronization. The tail behavior for them (power-law with a possibly different exponent), however, is inherited from the noise. Hence, the statistics of the extremes will be of the Fréchet type, yielding a power-law increase of the average size of the largest fluctuations above the mean.

The above picture is reasonably consistent in that the exponents for the tail behavior ($\sim 1/\Delta_i^{\mu+1}$) for both the probability density of the local heights $p(\Delta_i)$ and the extremes $p(\Delta_{\max})$ were within about 6%. Further, the average size of the extremes increases as $N^{1/\mu}$, in accordance with the underlying Fréchet distribution.

It is interesting to note that for $\mu \approx 2$, formally, $p(\Delta_i)$ does not have a finite

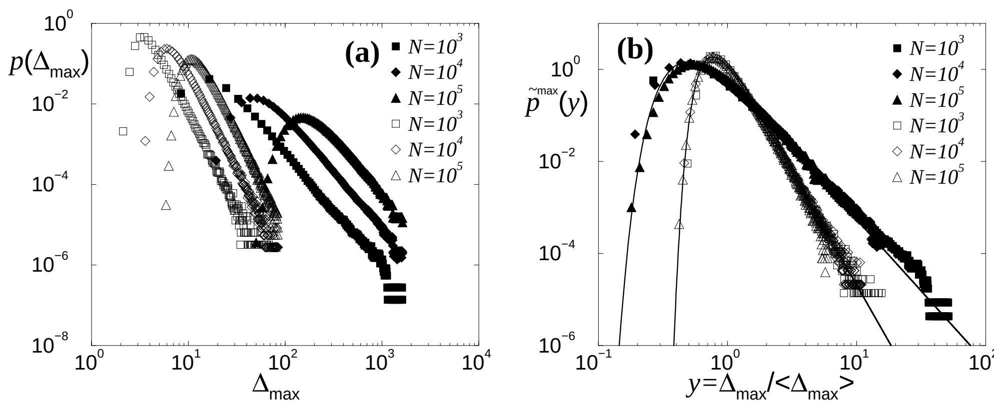

Fig. 8. (a) Disorder-averaged probability densities for the extreme-height fluctuations for the SW-synchronized (p=0.10) landscape in a power-law noise environment for γ=3 (filled symbols) and γ=5 (open symbols) for three system sizes indicated in the figure. Note the log-log scales. (b) Scaled probability densities. The solid curves correspond to the similarly scaled Fréchet density Eq. (11) for comparison.

variance (associated with the width $w = \sqrt{\langle(h_i - \bar{h})^2\rangle} = \sqrt{\langle(\Delta_i)^2\rangle}$). Indeed, in the simulations we observed large fluctuations in w and errorbars of the order of the width itself. The "theoretical" divergence for μ=2 is, of course, limited by the logarithm of a large but finite cutoff in the simulations. This anomalous (formally divergent) width is not related to a system-size dependent widening of the individual distributions controlled by a divergent correlation length. Rather, the individual distributions develop a heavy-tailed shape independent of the system size.

Examining the largest fluctuations below the mean reveals that they increase only logarithmically with the system size. This is simply the result of the exponential-like tail of the individual local height fluctuations below the mean [Fig. 6], where the governing limit distribution is of the Gumbel type (Sec. 2.1).

The above results show, that SW synchronization can be efficient to control the *average* size of the fluctuations, but the largest fluctuations still diverge in a power-law fashion with the number of nodes. While the SW-network effectively decouples the fluctuations in the synchronization landscape, it cannot suppress power-law tails already present in local noise distribution. In fact, the inherited power-law tails for the local height fluctuations are even "heavier" than that of the corresponding noise, $\mu\approx2$ for γ=3 and $\mu\approx4$ for γ=5.

6. Summary

We considered the extreme-height fluctuations in a prototypical model with local relaxation, unbounded local variables, and in the presence of exponential or power-law tailed noise. We showed that when the interaction topology is extended to include random links in a SW fashion, the local height variables become effectively independent and the statistics of the extremes is governed by the FTG or the Fréchet distribution, respectively. For both types of noise, the average width of the synchronization landscape becomes independent of the system size. The extreme fluctuations increase only logarithmically with the number of nodes for exponential-like noise and in a power-law fashion for the power-law noise. These findings directly addresses synchronizability in generic SW-coupled systems where relaxation through the links is the relevant node-to-node process and effectively governs the dynamics. We illustrated our results on an actual synchronizational problem in the context of scalable parallel simulations.

From a broader statistical physics viewpoint, the lines of investigations we pursue contribute not only to scalability and synchronizability, but also to general studies of collective phenomena on complex networks [1, 2, 15], e.g., on SW networks [12, 14, 39, 46–48, 87–96], and on scale-free [3, 97–104] networks. In particular, there is growing evidence that systems without inherent frustration exhibit (strict or anomalous) [39, 96] mean-field-like behavior when the original short-range interaction topology is modified to a SW network [39, 88–90, 92–96]. In essence, the (quenched) SW couplings, although sparse, induce an effective relaxation to the mean of the respective local field variables, and in turn, the system exhibits a mean-field-like behavior [39, 96]. This effect is similar to those observed in models with "annealed" long-range random couplings [105, 106].

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